(US) JOURNAL OF ADVERTISING RESEARCH

OCTOBER 1970

A new method is presented for estimating audience reach and frequency distribution which is relatively simple and low-cost yet better than many existing approaches.

Personal Media Probabilities

Jerome D. Greene

This article presents the personal probability method of extending the audience "reach" and "frequency distribution" of media advertising schedules, from survey data. First, however, it is necessary to review old methods.

One-Dimensional Case

One approach is the one-dimensional case. This is a schedule of multiple insertions in but one program, time period, newspaper, or magazine —for example, eight issues of a given magazine in a given population segment. Starting with survey readership out of four issues, for instance, how may this be extended to eight issues or any other number? Table 1 illustrates the problem.

On the left side of Table 1 are the tabulated survey findings for four issues, expressed in two ways. "Frequency" shows the number of people reading zero, one, two, three, or all four of the survey issues.

Easily derived from this frequency distribution is the corresponding survey "Reach," which shows the net audience cumulated by each number of issues up to four.

But suppose there is advertising in eight issues of this magazine. This constitutes a one-dimensional schedule of eight insertions. How can the survey data at the left for four issues be extended to the schedule data at the right for eight issues?

TABLE 1

THE PROBLEM IN ONE DIMENSION

Number of Issues	Survey: Reach %	Survey: Frequency %	Schedule: Reach %	Schedule: Frequency %
0	0.0	73.0	0.0	67.4
1	15.2	10.0	15.3	8.6
2	21.0	6.0	21.0	5.0
3	24.5	5.0	24.4	3.8
4	27.0	6.0	26.9	3.1
5			28.7	2.8
6			30.2	2.7
7			31.5	2.8
8			32.6	3.7
Total		100.0		100.0

There are several methods for doing this: actuarial, empirical, Methringham, beta, and personal probability. Each method estimates a person's probability "P" of exposure to an average unit of each medium, and then extends or projects in time his frequency of exposure to the number of media units in the schedule.

All of these methods extend in the same way, by the binomial theorem, and the differences and controversy lie not in how the extensions are made, but in how people's unit probabilities "P" are estimated from the survey to begin with. As an example, suppose it is known that for a certain group of people in the survey sample, their long-run probability of exposure to an average issue of a magazine was ¼. That is, as the number of issues is increased indefinitely, their limiting relative frequency or proportion of issues read converges on ¼.

What, then, is their frequency distribution out of, say, eight issues—their chance of reading none, one, two, three, . . . up to all eight issues? This is a simple binomial expansion as illustrated in Figure 1.

FIGURE 1

BINOMIAL THEOREM

Formula:

$$R_r^t = C_r^t \cdot P^r (1 - P)^{t-r}$$

Where: R_r^t = proportion exposed to "r" units out of "t"

C_r^t = combinations of "t" things "r" at a time

P = probability of exposure to one average unit

Example: $P = \frac{1}{4}$ issues read in long haul

$t = 8$ issues in schedule

For $r = 2$, proportion reading exactly 2 of 8 schedule issues equals:

$$R_2^8 = C_2^8 \cdot (\tfrac{1}{4})^2 \cdot (\tfrac{3}{4})^6 = 31.2\%$$

Therefore, of all people with ¼ probability, 31.2% will read 2 issues out of 8.

Figure 1 shows how *all* methods under discussion extend in time from survey to schedule. Of course, not all people have the same probability for each medium, and this is where methods differ. The problem, therefore, is estimating from the survey how many people have each possible probability value for each medium. Having done so, the binomial expansion is simply made separately for the people with each probability value, and added up.

Actuarial Probabilities

Some probability distribution must be assumed, and the obvious one is to say that each person has a unit exposure probability equal to the proportion of units he saw or heard in the survey.

This may be called the method of actuarial probability because the assumed probabilities are restricted to observed behavior points, as originally used in the insurance industry.

The people reading none of four issues in the example are considered to have zero probability and thus to read none of the later eight issues either. The people reading all four survey issues are considered to have a probability of one (certainty) and thus to read all eight schedule issues as well.

The in-between survey groups are extended by the binomial. For example, readers of one of the four survey issues, raising (¼ + ¾) to the 8'th power, are extended to give the proportions reading zero, one, two, three . . . up to all eight schedule issues, as illustrated.

Repeating across other survey frequency classes and adding up gives the estimated frequency distribution for eight schedule issues.

A necessary test of any extension method is how well it reproduces the survey data. But what happens if the same actuarial method is used to estimate the frequency distribution of only a four-issue schedule, the same as the survey? Table 2, which gives this comparison, shows that actuarial probabilities are wrong in the sense that they are inconsistent with the survey data.

For instance, it was said that a person reading none of four survey issues will never read an issue. This is like saying a person who hasn't died yet will never die. At the other extreme, it was said that a person reading all four survey issues will read all issues. This is like saying a person who has won four times in a row at the horse races will win forever.

TABLE 2

AUDIENCE REACH AND FREQUENCY ESTIMATES FOR 4 UNITS (PER CENTS)

	Actuarial Average Probability		*Actual Survey Data*	
No. Units	*Reach*	*Frequency*	*Reach*	*Frequency*
0	0.0	76.6	0.0	73.0
1	15.3	6.0	15.2	10.0
2	19.6	5.4	21.0	6.0
3	22.0	4.1	24.5	5.0
4	23.4	8.0	27.0	6.0
Total		100.0		100.0

The actuarial probabilities are not right even for the inbetween survey frequencies. It must be presumed, then, that people do not neatly conform their reading and listening and viewing rates to convenient fractions of the number of media units researchers chose to observe.

Why discuss a method that is so clearly wrong? Because it is still regularly used. For instance, the April 1970 issue of the *Journal of Marketing* has an article on a new media model computer program:

> The respondent was asked to indicate . . . whether he read all the issues, three-quarters of all issues, one-half of all issues, one-quarter of all issues, less than one-quarter, or none. . . .
>
> If one knew that a prospect was exposed to three out of four issues then the probability of exposure to the average issue would be .75; if the exposure was two out of four the probability would be .50; and so on.

The computer program then processes these actuarial probabilities as described, to estimate the reach and frequency distribution of advertising schedules.

As shown, such a method is badly biased, and produces misleading answers.

Empirical Probabilities

If one must use average or point probabilities, better ones can be found. Equations can be written to solve backwards for a set of point probabilities which will exactly reproduce the survey data.

This is called the method of empirical probabilities because whatever point probabilities work are taken empirically. Using the same example as before, Table 3 gives the empirical probabilities which reproduce the survey data.

TABLE 3

EMPIRICAL PROBABILITIES (PER CENT)

	All People	Number of Issues Read Out of 4: 0	1	2	3	4
"Actuarial"	15.2	0.0	25.0	50.0	75.0	100.0
"Empirical"	15.2	1.3	24.5	48.5	65.9	94.0

The only point probabilities that will reproduce the survey data require assigning an average greater than zero to readers of no survey issue, and less than one to readers of all survey issues. Certainly this makes sense.

By definition, this empirical probability method reproduces the survey exactly.

Continuous Probability Distribution

It must finally be presumed, however, that different people have different probabilities of exposure to a given unit of a medium on a continuous scale from zero to one. This does not preclude many people bunching up, however, in certain probability ranges.

In the past, the shapes of this distribution for specific media could be shown, thanks mainly to surveys sponsored by *Life Magazine* which covered six units each of many print and broadcast media and, in one famous case, 13 issues of *Life Magazine* itself.

The whole point of the well-known beta function is to express this distribution of people by their probabilities or rates of exposure.

The beta distribution is the only two-parameter, true probability function that can be concave up or down, and skewed left or right, thus reflecting the unique distribution of population probabilities of exposure to each individual media vehicle.

TABLE 4

AUDIENCE REACH AND FREQUENCY ESTIMATES FOR FOUR UNITS (PER CENTS)

No. Units	Actuarial Average Probability: Reach	Freq.	Empirical Average Probability: Reach	Freq.	Beta Probability Distribution: Reach	Freq.	Actual Survey Data: Reach	Freq.
0	0.0	76.6	0.0	73.0	0.0	73.1	0.0	73.0
1	15.3	6.0	15.3	10.0	15.3	9.7	15.2	10.0
2	19.6	5.4	21.0	6.0	21.0	6.1	21.0	6.0
3	22.0	4.1	24.5	5.0	24.4	5.1	24.5	5.0
4	23.4	8.0	27.0	6.0	26.9	5.9	27.0	6.0
Total		100.0		100.0		100.0		100.0

Jerome D. Greene is a founder and principal of Marketmath, Inc., which designs and applies mathematical-statistical methods to business problems in marketing and communications. He received an A.B. from Harvard and did graduate work in statistics at New York University. Through 1963, he was president and principal of Alfred Politz Media Studies and vice-president of the parent firm, Alfred Politz Research, Inc. He resigned to start his own marketing research business. Mr. Greene is the author of numerous published reports of media research methods and findings, and of various journal articles. He is a member of the Market Research Council, Modellers Discussion Group, American Statistical Association, and American Association for Advancement of Science.

Moreover, it works. Applied to only two or four units, the beta distribution method estimated audiences of six or 13 units which almost exactly matched the observed results of former studies which actually interviewed each respondent on that many units.

The beta distribution has two parameters, or mathematical constants, which control its shape. Of course, these are not known for a particular medium until it is surveyed. Then the values are found which give the best fit to the survey data, for each medium in each population segment.

Given these parameters, the survey frequency distribution can easily be extended to any desired number of media units. The extension formula simply amounts to doing the binomial expansion separately on the people at each probability value throughout the whole probability distribution.

Now results can be compared for all three methods. Table 4 does this for a four-issue schedule corresponding to the survey.

The actuarial method is bad. The empirical method fits the survey exactly because it was forced to. But the beta distribution method fits almost as well, and it is the only method that does not force people into discrete probability slots which are artifacts of the survey.

Table 5 shows the three methods extended to eight issues It is here that the empirical method begins to diverge from the beta distribution method, and becomes suspect of over-reaching while showing too many people exposed exactly once—a critical group for advertising evaluation.

TABLE 5

AUDIENCE REACH AND FREQUENCY ESTIMATES FOR EIGHT UNITS (PER CENTS)

No. Units	*Actuarial Average Probability* Reach	Freq.	*Empirical Average Probability* Reach	Freq.	*Beta Probability Distribution* Reach	Freq.
0	0.0	74.0	0.0	66.8	0.0	67.4
1	15.2	2.9	15.3	9.9	15.3	8.6
2	19.6	3.8	21.0	4.3	21.0	5.0
3	22.0	3.5	24.5	3.8	24.4	3.8
4	23.4	2.9	27.0	3.4	26.9	3.1
5	24.4	2.6	28.9	2.9	28.7	2.8
6	25.1	2.3	30.6	2.4	30.2	2.7
7	25.6	1.5	32.0	2.8	31.5	2.8
8	26.0	6.5	33.2	3.9	32.6	3.7
Total		100.0		100.0		100.0

TABLE 6

BETA DISTRIBUTIONS BY PROBABILITY OF EXPOSURE (PER CENTS)

Prob. Class	All People	*Number of Survey Units Exposed Out of 4* 0	1	2	3	4
0- 5	64.3	86.1	13.4	0.9	0.0	0.0
5- 10	5.8	5.9	13.7	2.7	0.2	0.0
10- 15	3.7	3.0	12.6	4.3	0.5	0.0
15- 20	2.8	1.8	11.2	5.7	1.0	0.0
20- 25	2.3	1.1	9.8	6.8	1.6	0.1
25- 30	2.0	0.7	8.4	7.6	2.3	0.2
30- 35	1.7	0.5	7.1	8.1	3.1	0.3
35- 40	1.6	0.3	5.9	8.4	4.0	0.5
40- 45	1.4	0.2	4.8	8.4	4.9	0.8
45- 50	1.3	0.1	3.8	8.2	5.8	1.2
50- 55	1.3	0.0	2.9	7.7	6.8	1.6
55- 60	1.2	0.0	2.2	7.1	7.6	2.2
60- 65	1.2	0.0	1.6	6.3	8.3	3.0
65- 70	1.1	0.0	1.1	5.4	8.9	4.0
70- 75	1.1	0.0	0.7	4.4	9.2	5.3
75- 80	1.1	0.0	0.4	3.4	9.3	7.0
80- 85	1.2	0.0	0.2	2.4	8.9	9.2
85- 90	1.2	0.0	0.0	1.5	8.0	12.3
90- 95	1.4	0.0	0.0	0.7	6.4	17.4
95-100	2.3	0.0	0.0	0.1	3.2	34.6
Total	100.0	100.0	100.0	100.0	100.0	100.0
Avg. Prob.						
Beta	15.2	2.5	23.4	44.2	65.0	85.9
Empirical	15.2	1.3	24.5	48.5	65.9	94.0
Actuarial	15.2	0.0	25.0	50.0	75.0	100.0
Class Size						
Beta Fit	100.0	73.1	9.7	6.1	5.1	5.9
Survey	100.0	73.0	10.0	6.0	5.0	6.0

Separate Beta Distribution in Each Frequency Class

Table 6 shows in the "All People" column the probability distribution for the magazine example among the total target population, summarized into five percentage-point classes. Clearly, this probability distribution should somehow be related to the observed survey frequency distribution. The corresponding probability distribution was derived mathematically for each survey frequency class *separately*, and this is important for the personal probability method. See the five breakdown columns of Table 6, according to survey frequency.

Note how readers of no survey issues are concentrated at the top of their probability distribution, with low probabilities. Conversely, readers of all survey issues are concentrated at the bottom of their distribution, with high probabilities.

The "beta average probabilities" at the bottom of the table are the means of the corresponding probability distributions above. Note that they have a smaller range—do not discriminate quite as well between survey frequencies—as the empirical probabilities beneath them, discussed previously.

Multi-Dimensional Case

If the only problem were extending one-medium schedules, the beta distribution method would be used, and it would be done with. A cheap desk computer will extend survey data to the reach and frequency distribution of any number of units in a minute.

Today's problems and debates center around optimal methods for extending the reach and frequency distribution of schedules using *combinations* of media. For a survey measuring

A γ .520 τ 1.08333 α .80166 P .281666

B γ .426 τ .74216 α .49553 P .24862

two issues, Figure 2 illustrates survey data for two magazines together.

Figure 2 shows every possible reading frequency combination. This matrix is summarized, as shown, to get the frequency distribution of "2-2" schedule—that is, using two issues of each magazine. The schedule frequency summary is usually the only thing needed, and it alone will be used.

Methringham Method

The question arises as to how is this extended to a schedule using more than two issues of one or both magazines. There is a simplistic method which ducks the whole problem by averaging media together. This creates a composite average medium which is then extended by the beta distribution. This is done by first converting the frequency matrix to its corresponding reach or cumulation matrix (Table 7), which shows the net audience cumulated by each combination of issues of the two magazines.

FIGURE 2

THE PROBLEM IN TWO DIMENSIONS

Magazine "A" Number Issues Read Out of 2	*Magazine "B" Number Issues Read Out of 2* 0	1	2	*"A" Marginal*
0	40	11	13	64
1	10	5	5	20
2	7	3	6	16
"B" Marginal	57	19	24	100%

SCHEDULE FREQUENCY SUMMARY

Number Issues Read Out of 4	*Per Cent of People %*	
0	40	(40)
1	21	(11 + 10)
2	25	(13 + 5 + 7)
3	8	(5 + 3)
4	6	(6)
Total	100	
"Reach"	60	(100 − 40)

TABLE 7

CUMULATION MATRIX

Magazine "A" Number of Issues	*Magazine "B" Number of Issues* 0	1	2
0	—	33.5	43.0
1	26.0	48.3	55.0
2	36.0	54.5	60.0

Consider extending by this averaging method the frequency distribution of a "4-4" schedule—that is, using four issues of Magazine A and also four of Magazine B. Table 8 shows how to take separate weighted averages of the one-issue and two-issue audiences, where the weights relate to the number of insertions planned in each vehicle. In calculating the average one-issue audience, the weight for each magazine's one-issue audience is the number of its issues to be used in the "4-4" schedule—that is, four.

In calculating the average two-issue cumulative audience, the weight for each magazine's two-issue cumulative audience is the number of possible pairs of two issues that can be formed from the number of its issues to be used in the schedule—that is, six. And the weight for the net audience of one issue of each magazine is the number of possible pairs of two issues, one from each magazine, in the "4-4" schedule—that is, 16.

This collapses the survey matrix down to two points, the one-issue and the two-issue cumulative audiences of a sort of composite medium in which it is planned to make eight insertions (4 + 4). Now it is easy to fit the beta distribution to these two average points and extend to eight issues, just as though this were a one-dimensional problem.

This is the Methringham method, devised by the British researcher Richard Methringham. The description in this article is far different and simpler than Methringham's, but it is algebraically identical and, it produces identical results.

The Methringham method has several problems when applied to U.S. media of overlapping geographic distribution. Media with different cumulation rates are each represented by the average cumulation rate among all of them. Pairs of media are each represented by the average net audience among them all. Beyond this, the average rates of cumulation within media and between media are further averaged together.

The Methringham method cannot reproduce U.S. survey data. Table 9 shows how it compares with the survey, using the two-magazine example, based upon the summary frequency distribution of the survey matrix. The schedule reach is somewhat off and the frequency distribution is way off. Due to combining media by averaging, the Methringham method cannot be right on the frequency distribution.

This bias is of acute concern for answering questions such as how many people will the schedule reach at least twice or three or four times . . . whatever threshold is right for the advertising strategy and copy.

TABLE 8

COMBINING TWO MAGAZINES

"A"	*"B"*	*Cumulative Audience %*		*Weight*		
0	1	33.5	×	4	=	134.0
1	0	26.0	×	4	=	104.0
				8		8 | 238.0
Average One-Issue Audience						29.8
0	2	43.0	×	6	=	258.0
1	1	48.3	×	16	=	772.8
2	0	36.0	×	6	=	216.0
				28		28 | 1246.8
Average Two-Issue Cumulative Audience						44.5

	Pub A	Pub B		IND	Base	Result
			0	.3648	.5127	.40
0	.5414	.4840	1	.2156	.1491	.2150
1	.16044	.1376	2	.2828	.1067	.2409
2	.11001	.10548	3	.0784	.09841	.0832
3	.09296	.10386	4	.3380	.1330	.0609
4	.0951	.1709		.6352	.4873	

Beta Matrix Method

The best way yet known of extending multi-media schedules may be called the matrix method because it uses all the information in the survey matrix or cross-tabulation of vehicles in the schedule, without averaging. Each row and column of the survey frequency matrix is separately fit and extended by the beta distribution, treating it as a distinct population segment—which indeed it is.

Table 10 shows how the matrix method reproduces the survey data in the previous example, compared with the Methringham method, when the frequency distribution of a "2-2" schedule is summarized corresponding to what the survey measured.

As in this example, the matrix method always reproduces the survey data very closely. Note in particular how it reproduces the peak at two exposures in the summary frequency distribution, which the Methringham method erroneously smooths out completely.

The matrix method is widely regarded and regularly used by many companies, including Marketmath, Simmons, Reader's Digest, and Time, Inc.

But there is a problem, mechanical and three-fold: First, the matrix method first requires pre-tabulating the survey to get the complete survey matrix or *m*-dimensional cross-tabulation of media vehicles in the schedule. Second, it then takes a big, fast computer to extend big schedules. Third, even so, the method cannot handle schedules of more than about six or eight vehicles or dimensions, depending on the number of intended insertions in each.

Also, the extended schedule matrix should be adjusted or "balanced" at the end for complete consistency with separate marginal extensions of each media vehicle alone. This takes further computation.

TABLE 9

SUMMARY FREQUENCY DISTRIBUTION

Schedule: 2 Units of Magazine "A"
2 Units of Magazine "B"

Number of Units Exposed in Schedule	*Methringham Method %*	*Actual Survey Data %*
0	38.4	40.0
1	26.5	21.0
2	18.3	25.0
3	11.5	8.0
4	5.4	6.0
Total	100.0	100.0
"Reach"	61.6	60.0

Personal Probability

With this background and definitions, a new approach is presented which shows promise of being simpler and better than the Methringham method—although not as good as the matrix method.

This new approach is called "Personal Probability." The logic is to pre-assign personal probabilities to each survey respondent on each medium, and to calculate schedule frequency distributions directly from these probabilities, processing each respondent in turn and finishing completely with one respondent before taking up the next.

Admittedly, it takes time to process each of 20,000 or whatever number of respondents, or a sample thereof. But present methods require preliminary cross-tabulations in any event, while the matrix method further requires processing a matrix which for large schedules contains many more cells than the sample size.

To illustrate this approach, Table 11 gives the beta probability distributions for Magazine "B" of the two-magazine example.

A table like this is in effect computed for every medium, by population segment. Each person is assigned a personal probability for each medium, randomly selected from the particular distribution corresponding to his reported exposure frequency in the survey. The random selection is arranged so that, across the whole sample segment, the assigned probabilities will distribute precisely as the computed distributions from which they were chosen.

Assigning personal probabilities separately by frequency class is the vital step which helps preserve intermedia correlations. The correlations are not perfectly maintained, however, as will be shown.

Personal probabilities may thus be assigned to all respondents on all media, once-and-for-all each year or whenever the new survey comes out, and saved on computer tape or disc. To calculate the reach and frequency distribution of a schedule, simply apply the binomial to each respondent in turn in the target population, for each medium in turn in the schedule.

Each respondent has a binomial frequency distribution for each media vehicle, expanded or extended to the number of units or insertions planned

TABLE 10

SUMMARY FREQUENCY DISTRIBUTION

Schedule: 2 Units of Magazine "A"
2 Units of Magazine "B"

Number of Units Exposed in Schedule	*Methringham Method %*	*Matrix Method %*	*Actual Survey Data %*
0	38.4	40.0	40.0
1	26.5	21.0	21.0
2	18.3	25.0	25.0
3	11.5	8.0	8.0
4	5.4	6.0	6.0
Total	100.0	100.0	100.0
"Reach"	61.6	60.0	60.0

in that vehicle. Summarizing each respondent's distributions, his personal contribution to the schedule frequency distribution is found. Each respondent is processed completely with minor demand on computer storage, before going to the next respondent.

This approach can handle vast 20-dimensional schedules such as annual media plans for combinations of print and broadcast, or many daytime TV shows in combination. The personal-probability assignment gets around problems of TV diaries covering more days than issues covered of magazines. No preliminary tabulations are required.

How well does it work? A final purpose of the personal probability approach is to avoid survey cross-tabulations of media combinations. For *testing* the method, however, such cross-tabs can be simulated to see how well they can be reproduced. In this case, the expected result of personal probability can be directly computed.

Table 12 shows how each multimedia method matches the survey data in the two-magazine example, on the frequency distribution of a "2-2" schedule corresponding to the survey. As shown previously, the matrix method reproduces the survey exactly, and provides a standard when extended to larger schedules.

Personal probability and Methringham are about the same on reach, but personal probability is much better on frequency.

Table 13 gives results for extending a "4-4" schedule—four issues of each magazine, or a total of eight insertions. Compared with the matrix method, personal probability is again about the same as Methringham on reach, and far better on frequency. Yet personal probability is easier and cheaper to do for large schedules.

The personal probability approach does not, as yet, maintain all the positive correlation between these two media, but improvement is possible. Table 11 shows the beta probability distributions for one of two magazines. This is how the personal probabilities were assigned for Magazine "B". Compare the beta average and empirical probabilities by frequency class, at the bottom of the table. Notice that the empirical probabilities spread out or discriminate more, as pointed out before.

One would achieve greater discrimination between frequency classes within each medium, and thus maintain more of the correlations between media, if the beta probability distributions were brought closer to centering on these empirical point probabilities. A tiny bias can be accepted in extending each media vehicle alone, for more accuracy in extending combinations of vehicles.

This optimization problem is the final detail to settle of the personal probability method, which should then be the optimal way of extending the reach and frequency distribution of large advertising schedules using many insertions in many print and/or broadcast media.

TABLE 11

BETA DISTRIBUTIONS BY PROBABILITY OF EXPOSURE (PER CENTS)

Prob. Class	*All People*	*Number of Survey Units Exposed Out of 2* 0	1	2
0- 5	36.2	62.3	3.7	0.0
5- 10	7.1	10.6	5.0	0.2
10- 15	4.9	6.6	5.5	0.3
15- 20	3.9	4.6	5.9	0.5
20- 25	3.3	3.5	6.1	0.7
25- 30	2.9	2.7	6.2	0.9
30- 35	2.7	2.2	6.2	1.2
35- 40	2.5	1.7	6.2	1.5
40- 45	2.4	1.4	6.1	1.8
45- 50	2.3	1.1	6.0	2.2
50- 55	2.2	0.9	5.9	2.6
55- 60	2.2	0.7	5.7	3.0
60- 65	2.2	0.5	5.5	3.6
65- 70	2.2	0.4	5.2	4.3
70- 75	2.3	0.3	4.9	5.1
75- 80	2.4	0.2	4.5	6.1
80- 85	2.6	0.1	4.0	7.5
85- 90	3.0	0.0	3.4	9.6
90- 95	3.7	0.0	2.7	13.4
95-100	8.9	0.0	1.5	35.6
Total	100.0	100.0	100.0	100.0
Avg. Prob.				
Beta	33.5	9.1	45.5	82.0
Empirical	33.5	4.9	47.6	90.3
Actuarial	33.5	0.0	50.0	100.0
Class Size				
Beta Fit	100.0	57.0	19.0	24.0
Survey	100.0	57.0	19.0	24.0

TABLE 12

SUMMARY FREQUENCY DISTRIBUTION

Schedule: 2 Units of Magazine "A"
2 Units of Magazine "B"

Number of Units Exposed in Schedule	*Methringham Method* %	*Matrix Method* %	*Personal Probability Method* %	*Actual Survey Data* %
0	38.4	40.0	38.0	40.0
1	26.5	21.0	22.8	21.0
2	18.3	25.0	26.0	25.0
3	11.5	8.0	8.4	8.0
4	5.4	6.0	4.8	6.0
Total	100.0	100.0	100.0	100.0
"Reach"	61.6	60.0	62.0	60.0

TABLE 13

SUMMARY FREQUENCY DISTRIBUTION

Schedule: 4 Units of Magazine "A"
4 Units of Magazine "B"

Number of Units Exposed in Schedule	*Methringham Method* %	*Matrix Method* %	*Personal Probability Method* %
0	27.4	30.9	27.8
1	18.0	12.9	15.3
2	14.0	11.7	12.7
3	11.4	12.2	12.4
4	9.3	16.2	16.2
5	7.5	5.8	6.2
6	5.8	3.7	4.2
7	4.2	2.9	3.1
8	2.4	3.6	2.2
Total	100.0	100.0	100.0
"Reach"	72.6	69.1	72.2